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Literature Reviews on Modeling Internal Geometry of Textile Composites and Rate-Independent Continuum Damage

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Preface

Textile composite materials have good potential for constructing composite structures where the effects of three-dimensional stresses are critical or geometric complexity is a manufacturing concern. There is a recent interest in advancing competence within LaRC for modeling the degradation of mechanical properties of textile composites. In an initial effort, two critical areas are identified to pursue:

- Construction of internal geometry of textile composites
- Rate-independent continuum damage mechanics

If a micromechanical approach is chosen for modeling the degradation of properties, consistent definition of the volumes taken by the fiber tows inside a unit cell of a textile composite are needed before a micromechanical analysis of the material can start. On the other hand, textile composites usually exhibit degradation of mechanical properties before macroscopic cracks appear. This phenomenon can be dealt with by applying continuum damage mechanics at either the macroscopic level or the microscopic level. A complete micromechanical modeling of the degradation of textile composites additionally requires techniques that resolve discrete cracks, e.g., cohesive finite elements, which have been investigated and implemented by researchers at LaRC, among others.

This memorandum documents reviews on the above two subjects. Various reviewed approaches are categorized, their assumptions, methods, and progress are briefed, and then critiques are presented. Each review ends with recommended research.

A Review on Approaches to Constructing Internal Geometry of Textile Composites

1. Introduction

The feasibility of finite element damage analysis of unit cells of textile composite materials is debilitated by the geometric complexity of the internal material interface. The internal geometry of complex fabrics generated by the existing modeling methods in conjunction with all of the input geometric data (e.g., weave type and tow spacing) determined “prior to fabrication” is either substantially idealized or very expensive to obtain. It is tempting to create internal geometry that is suitable for the geometry-based meshing paradigm to create a finite element mesh that is conformal with the material interface. (Such a mesh is referred to as a “conformal mesh” below.) Up to the present, this undertaking is only possible by substantial idealization and even assuming substantial idealization, the task is usually arduous. Most finite element modeling of unit cells of fabric composites in the literature falls into this category. The difficulty arises from the contact among the fiber tows, which is ubiquitous in actual textile composites, and the complexity and large aspect ratio of the volumes taken by the matrix. Therefore, one may attempt to develop a special meshing algorithm to obtain a conformal mesh, e.g., the method proposed by Kim and Swan (2003b), to convert a voxel-based mesh to a conformal mesh. On the other hand, one may attempt to develop generalized finite element approaches that use a non-conformal mesh, e.g., the voxel-based mesh refinement (Kim and Swan, 2003a), the heterogeneous finite element analysis (Wang, 2003), the structured extended finite element method (Belytschko, Parimi, et al., 2003), the mesh superposition method (Nakai, Kurashiki, and Zako 2007), and the independent mesh method (Iarve, 2008).

Whatever roadmap for achieving a finite element solution for the unit cell of a textile composite is taken, the first step is to build a geometric model that provides consistent definitions of the volumes taken by the fiber tows inside the unit cell. The present review first summarizes approaches for creating the internal geometry of textiles that do not involve simulation of the mechanical interaction among the fiber tows, and then presents approaches that involve simulation of the interaction.

2. Creating Internal Geometry

2.1. CAD-based Internal Geometry

The internal geometry of fabrics has been generated without simulating the mechanical interaction among the interlacing fiber tows. Therefore, the internal geometry is essentially generated by CAD. Research emphasis in this area has been placed on building automated CAD procedures and special CAD libraries to improve efficiency in modeling that is based on the paradigm of geometry-based conformal mesh generation (e.g., Wentorf, Collar, et al., 1999). Since the mechanical interaction among the fiber tows is not simulated, some input geometric data have to be determined empirically or experimentally.

Highly idealized geometric models in which each fiber tow has a constant elliptical or lenticular cross section have been employed in most finite element analyses of unit cells of fabric composites (e.g., Thom, 1999; Carvelli and Poggi, 2001; Whitcomb and Tang, 2001; Goyal and Whitcomb, 2006). Due to the strong idealizing assumption on geometry, these models require the smallest number of input geometric data as compared with other CAD-based geometric models.

Hivet and Boisse (2005) developed an automated CAD procedure to create parametric solid models for two-dimensional (2-D) weaves (plain weave, twill, and satin). In the procedure, CAD solids, representing the yarns, are generated by sweeping along the yarn trajectories and blending the cross sections of the yarns. Several assumptions are made in order to generate yarn-to-yarn areal contact, without causing interpenetration, and other experimentally observed geometric characteristics. The cross sections of the yarns are represented as four-sided planes. Each side can be either straight or curved (parabola or arc). A pair of opposite sides are not in contact with any other yarn and this pair may degenerate to two points if a two-sided cross section is desired. The cross sections of a yarn are enveloped by a constant rectangle specified for that yarn. The shape of a contact side of a yarn is identical to the shape of the trajectory of the other overlapping yarn in the contact zone and vice versa. The trajectory in a contact-free zone is assumed to be straight in view of the small bending stiffness. In addition, if a yarn consecutively crosses over m yarns on the same side, the contact sides of the 2nd, 3rd ..., ($m-1$)th yarns are straight in the contact zones. Asymmetry of a cross section about the vertical plane, which is orthogonal to the fabric plane and passes through the trajectory of the yarn, is allowed if symmetry cannot be inferred from the weave pattern. It is shown that the model requires three and seven input geometric data for balanced 2-D fabrics and unbalanced 2-D fabrics, respectively.

In the work of Crookston, Ruijter, et al. (2007), each tow volume is defined with its centerline and several cross sections taken at different locations along the centerline. The centerline and cross sections are defined with splines. Both contact and interpenetration among the fiber tows are avoided by automatic adjustment of the local cross sections. Accordingly, the local fiber volume fractions are adjusted as well in order to obtain a consistent total amount of reinforcement. It should be noted that without taking into account the mechanical interaction among the fiber tows, there is always excessive freedom in adjusting a cross section. Crookston, Ruijter, et al. (2007) pointed out difficulties in automating CAD processes for creation of the internal geometry that includes contact among the fiber tows and is suitable for geometry-based conformal mesh generation. It should be noted that if contact is allowed, topology very probably changes from one tow to another and from one set of values of textile design parameters to another. In addition, creating geometry with complicated topology is prone to numerical problems. An example is creating intricate matrix volumes by performing boolean operations that involve contacting tows in a unit cell.

2.2. Internal Geometry Created by Simulating Mechanical Interaction among Fiber Tows

Lomov, Huysmans, et al. (2001) and Verpoest and Lomov (2005) presented a method, which has been implemented in a textile geometry modeling program (*WiseTex*), to determine the centerlines and the cross sections of the yarns that are affected by the mechanical interaction. The cross section of a yarn is assumed to be either elliptical or lenticular. The cross-sectional height and width are experimentally determined nonlinear functions of the compressive transverse force acting on the yarn. Bending and longitudinal shear deformation of the fiber tows are assumed to be elastic. It should be noted that in reality, elasticity never dominates shear deformation. In the method, each tow is divided into “crimp intervals,” each of which represents a section of the tow between interlacing sites. Each crimp interval is assigned a deflection trial function with one unknown, “crimp height,” representing the deflection at one end of the crimp interval relative to the other end. The trial function is a geometrically-nonlinear approximate solution for a beam subjected to shear loads and the no-rotation condition at the ends of the beam and is nonlinear in terms of the crimp height. The elastic bending energy stored in a crimp interval can be derived from the deflection function in conjunction with the experimentally

measured curvature-dependent bending rigidity, so that the bending energy is a nonlinear function of the crimp height. According to the geometrically-linear-theory, the bending energy stored in a crimp interval is equal to one half of the product of the shear load and the crimp height. By assuming that this relation holds for the energy derived from the geometrically-nonlinear deflection, the shear load on a crimp interval is expressed as a nonlinear function of the crimp height. Furthermore, the compressive transverse force exerted on a crimp interval by its interlacing tows is approximated by the shear load, so the cross-sectional height of a crimp interval becomes a nonlinear function of the shear load. Linear kinematic constraints in terms of the cross-sectional heights and crimp heights are set up to ensure deflection continuity for each tow and point-contact between interlacing tows. Essentially, these constraints can be reduced to nonlinear equations in terms of the crimp heights. Numerical iteration is performed between minimizing the total bending energy and satisfying the constraints in order to solve for the unknown crimp heights, from which all the other variables can be determined. Once the local cross sections are determined, the complete yarn geometry can be reconstructed by spline approximation. A very short computational time for a complicated unit cell has been reported. In the work of Lomov and Verpoest (2006), the above method is applied to simulate deformation of the yarns of dry fabrics subject to tension. Friction among the interlacing tows is incorporated into the method to simulate the response to shear loading. Such characterization is important to prediction of fabric draping. As discussed by Lomov, Ivanov, et al. (2007), the simplified yarn deformation and contact in the method may impair fidelity and result in interpenetration among fiber tows. Conceivably, the problem is serious for fabrics where some fiber tows are locally sandwiched by others or have substantially nonsymmetrical cross sections, e.g., dense fabrics, three-dimensional (3-D) fabrics, and fabrics with non-orthogonal yarns. An ad hoc finite-element based mesh deformation procedure for eliminating the interpenetration was presented by Lomov, Ivanov, et al. (2007). In addition to the ad hoc nature, the procedure has a limitation that it results in no contact between the fiber tows.

Zhou, Sun, and Wang (2004) developed the so-called multi-chain digital element approach to simulate textile fabrication (such as weaving and braiding) and forming in order to predict the internal geometry of textiles. A digital element is a one-dimensional (1-D) rod finite element (having only the axial stiffness) and a chain is a serial assembly of digital elements. In a multi-chain digital element model, each fiber yarn is modeled as a bundle of chains and node-to-node contact elements are dynamically added or removed to simulate frictional contact between chains. Thus, a textile fabrication or deformation process, being treated as a geometrically nonlinear static contact problem, can be analyzed by using an implicit finite element code. The analysis yields the movement of the chains relative to one another, which determines the final yarn geometry. Note that the number of chains used for a fiber yarn defines the upper bound for the cross-sectional aspect ratio of the deformed yarn. Step-by-step simulations of a 2-D weaving process and of a 3-D braiding process were demonstrated by Zhou, Sun, and Wang (2004) and simulation of vacuum bag compression of randomly nested layers of a 2-D braided preform was demonstrated by Zhou, Mollenhauer, and Iarve (2008). Excellent correlation between the internal geometry that is predicted by the 2-D weaving and the 3-D braiding simulations, and the experimentally observed geometry is shown in the work of Miao, Zhou, et al. (2008). Their numerical study also addresses a major concern of the computational time required for the simulation using the multi-chain digital element approach. It is shown that the predicted geometry for the numerical examples obtained by using about 20 chains per fiber yarn has quality similar to that obtained by using more chains. It is also shown that the time-consuming step-by-step simulation of the fabric processes in the examples is not necessary to obtain the final internal geometry. A much shortened simulation that starts with a specified interlacing pattern (topology) and fiber yarns having circular cross sections and omits the effect of the chain-to-chain friction yields nearly the same result. This finding suggests that the final internal geometry is primarily

determined by the interlacing topology, the initial cross-sectional areas of the yarns, and the boundary conditions. Detailed information about the model size and computational time is not provided in the paper. In a continuum mechanics analysis of a textile composite, each tow is modeled as a solid entity of a unidirectionally continuous fiber reinforced composite. Note that a bundle of deformed chains, modeling a deformed yarn, is not a solid; therefore, the bundle cannot be used directly to create a solid mesh for the deformed yarn. Zhou, Mollenhauer, and Iarve (2008) presented a procedure to construct a solid approximating a bundle of deformed chains so that a solid finite element mesh can be created subsequently. First, cross sections of a chain bundle are taken at various locations along the yarn path, including its two ends, and each cross section is approximated by a polygon. Then, the lateral surface of a yarn can be represented by a set of triangular facets which are obtained by connecting the vertices of the cross-sectional polygons. Finally, a solid can be defined with the lateral surface and appropriate boundaries of the unit cell. It should be noted that the resulting solid geometry has to be adjusted in order to eliminate interpenetration or to ensure well-defined contact among the fiber yarns.

Durville (2005) modeled a fiber with special 1-D large-strain beam finite elements connected in series. By assigning nine degrees of freedom to each node, the kinematic formulation of the beam element allows not only the axial strain, bending strains, and transverse shear strains but also the cross-sectional planar stains. If a bundle of fibers, instead of a single fiber, is modeled as a series of the beam elements, then a yarn can be modeled as a bundle of beam element series. Under the circumstances, the incorporation of the cross-sectional planar stains should conduce to a reduction in the number of beam element series needed to model the yarn. A “symmetric” contact-point search algorithm that uses the normal direction to an “intermediate geometry” as the search direction, instead of traditionally the normal direction to either one of two contact candidates, is employed in order to more accurately capture contact between two contact candidate intervals of high-curvature fiber-like structures. The intermediate geometry is defined as the geometric average of the contact candidate intervals. A desired number of contact elements per unit length of the intermediate geometry are created between the identified contact points on a “fiber” and those on the other contacting “fiber,” not necessarily between nodes. In general, Durville’s contact algorithm is an improvement from the multi-chain digital element approach. Durville (2007) ran an implicit static finite element code on a cluster of six processors to predict the internal geometry of a relaxed plain weave and its internal deformation in response to external shearing, bending, and twisting. In the work, each yarn is modeled with 28 series of the beam elements and the internal geometry is obtained without following the weaving kinematics. The CPU time spent on an analysis is reported to be a couple of days.

3. A Recommended Research Approach to Constructing Internal Geometry of Textile Composites Suitable for Damage Analysis

CAD-based internal geometry is adequate for predicting the effective properties of textile composites. Highly-idealized CAD-based geometric models are not suitable for the analysis of damage evolution, which is sensitive to local geometric details. For some 2-D fabrics, with relatively simple internal geometry, more realistic CAD-based models in conjunction with all of the input geometric data determined prior to fabrication may be sufficient for damage analysis. For fabrics with more complicated internal architecture, sophisticated CAD-based modeling is prone to robustness problems and involves input geometric data that have to be determined experimentally. Suitable internal geometry for damage analysis can be obtained by modeling the tows as bundles of series of 1-D finite elements in a contact analysis. However, the numerical implementation is too time-consuming for practical implementation, even if the step-by-step

textile process is omitted. A potential solution to this problem recommended herein is to model the tows, arranged according to the desired interlacing topology, with 3-D finite elements in a surface-to-surface contact analysis. A transversely isotropic elastoplastic material model is used for the tows and a pressure-dependent yield strength is used to simulate the effect of fiber-to-fiber friction. As suggested by Miao, Zhou, et al. (2008), each fiber tow may be assigned a constant cross section when building the initial geometry. It is recommended herein that suitable initial cross sections are chosen to avoid interpenetration and subsequently the cross-sectional areas are increased in a controlled manner during the simulation of contact. The controlled increase in cross-sectional area is achieved by using a fictitious coefficient of transverse thermal expansion for the tows. The proposed approach should have another time-saving advantage over the approaches using bundles of series of 1-D finite elements because solids are directly obtained from the proposed simulation.

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A Review on Rate-Independent Continuum Damage Mechanics

1. Continuum Damage Mechanics Applied to Four Categories of Material Response

There have been noticeable advancements in continuum damage mechanics (CDM) over the past two decades. Rate-independent CDM theories are intended to describe progressive loss of stiffness of a rate-independent material as a result of the loading-induced initiation, growth, and coalescence of distributed cracks “inside” the material until the discrete nature of dominating cracks cannot be ignored for a structural mechanical problem in hand.

CDM theories have been developed to model the effects of anisotropic damage on the elastic response of materials with initial isotropy (e.g., Chow and Wang, 1987; Valanis, 1990; Tang, Jiang, et al., 2002), the elastoplastic response of materials with initial isotropy (e.g., Ju, 1989; Chow and Chen, 1992; Wu and Nanakorn, 1999; Voyiadjis and Park, 1999), the elastic response of continuous-fiber reinforced composites (e.g., Talreja, 1991; Sørensen and Talreja, 1993; Maire and Chaboche, 1997; Chaboche and Maire, 2001; Varna, Joffe, et al., 2001; Williams and Vaziri, 2001; Talreja, 2005; Hassan, 2005; Camanho, Maimí, et al., 2007), and the elastoplastic response of continuous-fiber reinforced composites (e.g., Voyiadjis and Deliktas, 2000; Maa and Cheng, 2002).

The present review covers the above four types of damaged material response, and focuses only on the rate-independent material behavior.

2. Modeling Effects of Damage

Many CDM models incorporate the effects of damage by using the concept of effective stress, which is usually related to the nominal (macroscopic) stress through a single fourth-order damage tensor. For the case of isotropic damage, the fourth-order damage tensor reduces to a single scalar damage variable. Other models do not employ the concept of effective stress, but instead directly adjust the elastic moduli by using a set of scalar damage variables, or define damage in terms of micromechanical quantities.

2.1. Modeling Effects of Damage by Using the Concept of Effective Stress

A majority of CDM models either explicitly or implicitly employ the concept of effective stress, which is representative of the internal stress state of a damaged material. Damage causes reduction in the load-carrying cross-sectional area of material. The traction acting on a reduced cross section is calculated from the effective stress tensor by using Cauchy’s formula. It is usually assumed that the effective stress is a linear tensorial function of the nominal stress. Referring to a Cartesian coordinate system, this relation is written as follows:

$$\begin{aligned}\bar{\sigma}_{ij} &= M_{ijkl} \sigma_{kl} \\ \sigma_{ij} &= \Phi_{ijkl} \bar{\sigma}_{kl}\end{aligned}$$

where $\bar{\sigma}_{ij}$ is the effective stress tensor, σ_{kl} is the nominal stress, and M_{ijkl} is the damage effect tensor, and Φ_{ijkl} is the integrity tensor. The integrity tensor can be rewritten as

$$\Phi_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - D_{ijkl}$$

where D_{ijkl} is called the fourth-order damage tensor.

2.1.1. The Role of the Fourth-Order Damage Tensor

Some researchers directly used the fourth-order damage tensor, D_{ijkl} , as a model variable (e.g., Tang, Jiang, et al., 2002; Ju, 1989; Williams and Vaziri, 2001), although its microscopic geometric meaning is not clear in general. A special case was given by Tang, Jiang, et al. (2002), where D_{ijkl} is related to volume fractions of orthotopically distributed needle-shaped microvoids.

Many researchers (e.g., Chow and Wang, 1987; Valanis, 1990; Chow and Chen, 1992; Wu and Nanakorn, 1999; Voyiadjis and Park, 1999; Maire and Chaboche, 1997; Chaboche and Maire, 2001; Voyiadjis and Deliktas, 2000) employed heuristic construction of the fourth-order damage effect tensor (or damage tensor) from a symmetric second-order damage tensor which characterizes reduction in load-carrying cross-sectional area of material. Although the construction of the fourth-order damage effect tensor in these approaches is heuristic and it involves ad hoc symmetrization in order to ensure the symmetry of the effective stress, the second-order damage tensor is widely used because of its clear microscopic geometric meaning, which was described by Voyiadjis and Deliktas (2000) by constructing a fictitious undamaged configuration through coordinate transformation. In the work of Voyiadjis and Deliktas (2000), it is shown that the fictitious configuration has a reduced volume due to the elimination of damage. Jirasek (2007) pointed out that the formulations based on the symmetric second-order tensor are limited to orthotropic damage whose principal directions may change along the loading history.

2.1.2. Relating Damage to Mechanical Behavior

The elastic stiffness of a damaged material can be expressed as a function of the fourth-order damage tensor or its degenerates by adopting either the hypothesis of strain equivalence or the hypothesis of elastic strain energy equivalence.

2.1.2.1. The Hypothesis of Strain Equivalence

The effective-stress based CDM models developed by Tang, Jiang, et al. (2002), Ju (1989), Maire and Chaboche (1997), Chaboche and Maire (2001), and Williams and Vaziri (2001) are constructed with the aid of the hypothesis of strain equivalence proposed by Lemaitre and Chaboche (1978). The hypothesis states that the nominal stress applied to a damaged material produces the same strain as the effective stress applied to the fictitious undamaged material. This approach leads to a non-symmetric elastic stiffness tensor as follows:

$$C_{ijkl} = \Phi_{ijmn} C_{mnkl}^0 = C_{ijkl}^0 - D_{ijmn} C_{mnkl}^0$$

where C_{ijkl} and C_{ijmn}^0 are the damaged elastic stiffness and undamaged elastic stiffness, respectively. Therefore, ad hoc treatment has to be performed on C_{ijkl} to obtain the symmetry (Maire and Chaboche, 1997; Chaboche and Maire, 2001; Williams and Vaziri, 2001). Conceivably, an alternate approach is to directly use symmetric C_{ijkl} as the damage variable (Tang, Jiang, et al., 2002; Ju, 1989). It is worth noting that the hypothesis of strain equivalence leads to reductions, in the axial elastic moduli, proportional to the respective axial damages when the principal directions of D_{ijkl} are aligned with the material principal axes.

2.1.2.2. The Hypothesis of Elastic Strain Energy Equivalence

The effective-stress based CDM models in the works of Chow and Wang (1987), Valanis (1990), Chow and Chen (1992), Wu and Nanakorn (1999), Voyiadjis and Park (1999), and Voyiadjis and Deliktas (2000) are constructed with the aid of the hypothesis of elastic strain energy equivalence proposed by Cordebois and Sidoroff (1979). The hypothesis states that the density of strain energy stored in a damaged material subject to the nominal stress is equal to the density of strain energy stored in the fictitious undamaged material subject to the effective stress. This approach yields a symmetric elastic stiffness tensor. It is noted that the afore-mentioned volume reduction (Voyiadjis and Deliktas, 2000) does not enter the calculation of strain energy density and an anomaly may ensue. For example, consider uniaxial damage of a “material” consisting of a bundle of parallel elastic fibers subject to the nominal tensile stress σ . Let E denote the Young’s modulus of the intact material and d denote the fraction of broken fibers due to the applied load. Consequently the effective stress is $\sigma/(1-d)$ and the strain energy density of the undamaged material is $\sigma^2/2E(1-d)^2$. Using the hypothesis of strain energy equivalence yields the Young’s modulus of the damaged material being $E(1-d)^2$, which is less than the expected value $E(1-d)$. The latter coincides with the prediction by using the hypothesis of strain equivalence. It should be noted that the distinctive factor $1-d$ is exactly the volume fraction of the undamaged material.

2.2. Modeling Effects of Damage without the Concept of Effective Stress

Some CDM models directly characterize engineering elastic parameters without referring to the concept of effective stress (e.g., Camanho, Maimí, et al., 2007; Maa and Cheng, 2002). The CDM models in the papers of Camanho, Maimí, et al. (2007) and Maa and Cheng (2002) for unidirectional composite laminas subject to plane stress assume that the original principal axes of material are retained after damage, only the in-plane axial elastic moduli are reduced, and the reductions are proportional to the respective axial damages. Scalar damage variables can be defined solely for the purpose to facilitate formulation of the reductions.

It is possible to define damage variables in terms of micromechanical quantities. In the works of Talreja (1990, 1991), Sørensen and Talreja (1993), Varna, Joffe, et al. (2001), Talreja (2006), and Singh and Talreja (2008), strain-like second-order tensors are defined in terms of micromechanical quantities (such as crack density, crack-face displacement discontinuity per unit of applied macro-strain) for damage modes of matrix cracking normal to fibers, fiber-matrix interfacial debonding, and interfacial slip in unidirectional ceramic matrix composites, and through-the-ply-thickness transverse cracking in polymeric matrix composite laminas. In the

work of Hassan (2005), three micromechanical scalar damage variables are adopted to represent the fractions of ineffective broken fibers due to longitudinal loading, cracked matrix due to transverse loading, and fiber-matrix interfacial debonding due to longitudinal shear, respectively, in unidirectional AS4/PEEK composite laminae. A simple rule-of-mixtures micromechanical analysis is employed in the work of Hassan (2005) to derive relations between engineering elastic parameters and the scalar damage variables. The results show that only the in-plane axial elastic moduli are reduced and the reductions are nearly proportional to the respective damage variables.

3. Modeling Evolution of Damage

With the elastic stiffness expressed as a function of damage variables, CDM models characterize the reduction in the elastic stiffness by tracking the evolution of the damage variables as a result of a loading history. To this end, damage evolution equations must be proposed in any CDM theory.

3.1. Modeling Evolution of Damage within the Framework of Thermodynamics

Damage variables used for a macroscopic description of a damage process are indeed internal (or hidden) variables. To describe more complex material behavior, additional internal variables may be required. Like any other constitutive theory with internal variables, CDM models with internal variables must be formulated in a way such that the second law of thermodynamics is satisfied by any admissible change in the material state. Therefore, for such CDM models, the damage evolution equations must be derived within the framework of thermodynamics.

3.1.1. Clausius-Duhem Inequality and Internal Variables

With selected damage variables, CDM constitutive equations are usually derived from or devised to satisfy the Clausius-Duhem inequality. The formulation starts with postulating a functional form for the Helmholtz free energy or Gibbs free energy. If the material is elastic, a free energy function can be readily obtained with a known relation between the elastic stiffness (or compliance) and damage variables (e.g., Valanis, 1990; Tang, Jiang, et al., 2002; Maire and Carboche, 1997; Chaboche and Maire, 2001; Williams and Vaziri, 2001; Hassan, 2005; Camanho, Maimí, et al., 2007). If the material exhibits plastic behavior, the free energy also depends on additional internal plasticity variables that complement characterization of the plastic state, and may further depend on other internal damage variables that complement characterization of the damage state (e.g., Ju, 1989; Chow and Chen, 1992; Wu and Nanakorn, 1999; Voyiadjis and Deliktas, 2000; Maa and Cheng, 2002). For this case, the construction of a free energy function may be guided by the choice of the hypothesis of strain equivalence or strain energy equivalence (e.g., Ju, 1989; Chow and Chen, 1992; and Wu and Nanakorn, 1999). Having an assumed free energy function, one may proceed to identify the thermodynamic force conjugate to a damage or plasticity variable as the derivative of the free energy with respect to that variable.

3.1.2. Derivation of Damage Evolution Equations

Once the free energy, internal variables, and their conjugates are established, damage evolution equations are usually derived from either the hypothesis of maximum dissipation or the

so-called linear thermodynamic law. Both arguments are sufficient conditions for the Clausius-Duhem inequality.

3.1.2.1. Hypothesis of Maximum Dissipation

A majority of CDM models require a damage potential (damage criterion) as a function of the thermodynamic forces conjugate to the damage variables and if plasticity is involved, a plasticity potential (yield criterion) as a function of the thermodynamic forces conjugate to the plasticity variables. Then, the hypothesis of maximum dissipation under the constraint of a damage criterion and additionally under the constraint of a yield criterion, if plasticity is involved, may be invoked to obtain “flow rules,” which state that increments of model variables are proportional to gradients of the potentials with Lagrange multipliers as the proportionality factors. Finally, evolution equations are derived from combining the “flow rules” and consistency conditions, which are inferred from the damage and yield criteria (e.g., Tang, Jiang, et al., 2002; Ju, 1989; Maire and Carboche, 1997; Chaboche and Maire, 2001; Voyiadjis and Deliktas, 2000). The CDM models proposed by Williams and Vaziri (2001), Hassan (2005), Camanho, Maimí, et al., (2007), and Maa and Cheng (2002) for unidirectional composite laminas only take into account degradation of the in-plane axial elastic moduli. Therefore, for these cases, the above thermodynamics-based methodology merely leads to a condition that the damage variables are monotonically increasing along the loading history, and optional use of the thermodynamic force conjugates for formulating the damage criterion (or yield criterion) and evolution equations.

3.1.2.2. Linear Thermodynamic Law

Instead of the constrained maximization of dissipation, the linear thermodynamic law that the rate of change of a thermodynamic variable is proportional to the conjugate thermodynamic force can be used to devise evolution equations (e.g., Valanis, 1990; Chow and Chen, 1992; Wu and Nanakorn, 1999). From a theoretical standpoint, this concise approach does not require a damage criterion or yield criterion. A suitable equation governing evolution of a model variable can be contrived by judiciously implementing the linear thermodynamic law so that the dissipation due to an increment of the variable is positive definite.

3.2. Modeling Evolution of Directly Measurable Damage Variables

A polynomial expansion of the free energy is used in the works of Talreja (1990, 1991), where damage variables are defined in terms of micromechanical quantities. As dictated by classical linear elasticity theory, the polynomial expansion is quadratic in strain components. On the other hand, the researchers’ prior experience suggests that a linear expansion in the damage variables as well as products of some paired damage variables (accounting for coupled damage mechanism such as fiber-matrix interfacial slip in conjunction with matrix cracking) should be adequate. The damage-stress-strain relationship with indeterminate coefficients for the polynomial terms is derived by differentiating the free energy with respect to strains. In the subsequent works of Sørensen and Talreja (1993), Varna, Joffe, et al. (2001), Talreja (2006), and Singh and Talreja (2008), a procedure involving experimental measurement of micromechanical quantities (e.g., crack density) or a combination of experimental measurement and micromechanical analysis (e.g., crack-opening displacement) is used to evaluate the coefficients and evolution of damage variables for some simple loading conditions. Interestingly, the

procedure takes advantage of the micromechanical root of the damage variables and thus does not resort to the second law of thermodynamics in the macroscopic form.

4. A Recommended Research Approach to Modeling Degradation of Elastic Properties

It is well known that effective elastic properties of a continuum with periodically distributed micro-cracks can be computed with good accuracy by using a micromechanical model containing a small number of degrees of freedom. This suggests that the concept of effective stress in CDM could be generalized and incorporated into the common stress averaging scheme of micromechanics. Therefore, one might characterize damage with internal volumes (say, “ineffective volumes”) where stress is reduced due to degradation, and then proceed with micromechanical formulation, which is particularly useful to model inhomogeneous materials. The tensorial representation of damage is recommended not only because of its operational conciseness but also its ability to capture desired physical meaning. It should be noted that an ineffective volume has attributes such as size, shape, and orientation. The thermodynamic framework has proven to be a successful tool for modeling mechanical behavior of continua. From a perspective of developing constitutive models for the purpose of computational structural analysis, the thermodynamic approach is necessary as complicated interaction among micro-damages cannot be realistically or efficiently simulated merely by micromechanical analysis.

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